ELSEVIER

Nuclear Physics B441 (1995) 197–214

NUCLEAR PHYSICS B

QCD constraints on the shape of polarized quark and gluon distributions ☆

Stanley J. Brodsky [a], Matthias Burkardt [b,1], Ivan Schmidt [c]

[a] *Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA*
[b] *Center for Theoretical Physics, Laboratory for Nuclear Science, and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA*
[c] *Universidad Federico Santa María, Casilla 110-V, Valparaiso, Chile*

Received 8 February 1994; revised 29 December 1994; accepted 29 December 1994

Abstract

We develop simple analytic representations of the polarized quark and gluon distributions in the nucleon at low Q^2 which incorporate general constraints obtained from the requirements of color coherence of gluon couplings at $x \sim 0$ and the helicity retention properties of perturbative QCD couplings at $x \sim 1$. The unpolarized predictions are similar to the D'_0 distributions given by Martin, Roberts, and Stirling. The predictions for the quark helicity distributions are compared with polarized structure functions measured by the E142 experiment at SLAC and the SMC experiment at CERN.

1. Introduction

Measurements of polarization correlations in high momentum transfer reactions can provide highly sensitive tests of the underlying structure and dynamics of hadrons. The most direct information on the light-cone momentum distributions of helicity-aligned and helicity-anti-aligned quarks in nucleons is obtained from deep inelastic scattering of polarized leptons on polarized targets. Recent fixed-target measurements, including the CERN SMC muon–deuteron experiment [1], the electron–He^3 and electron–proton experiments E142 and E143 at SLAC [2], and the SMC muon–proton experiment [3] are now providing important new constraints on the proton and neutron helicity-dependent structure functions.

☆ Work supported in part by the Department of Energy, contracts DE-AC03-76SF00515 and DE-AC02-76ER03069, and by the Fondo Nacional de Investigación Científica y Tecnológica, Chile, contract 1931120.

[1] Present address: Institute for Nuclear Theory, University of Washington, Seattle, WA 98195, USA.

SSDI 0550-3213(95)00009-7

Although the Q^2-evolution of structure functions is well predicted by perturbative QCD, the initial shape of these distributions reflects the non-perturbative quark and gluon dynamics of the bound-state solutions of QCD. Nevertheless, it is possible to predict some aspects of the shape of the input nucleon structure functions from perturbative arguments alone. In this paper, we will develop simple analytic representations of the quark and gluon helicity distributions which incorporate general constraints obtained from the color coherence of the gluon couplings at $x \sim 0$, and the helicity structure of perturbative QCD couplings at $x \sim 1$. Since we work at the bound-state scale, we can directly impose global sum rules and symmetries such as the axial coupling constraint $\Delta u - \Delta d = g_A$, from neutron beta decay. The parameterizations we use have the minimal number of parameters needed to satisfy all of the constraints. The predicted forms for the quark and gluon helicity distributions, $\Delta q(x) = q^+(x) - q^-(x)$ and $\Delta G(x) = G^+(x) - G^-(x)$, should provide useful guides to the expected shapes of the polarized structure functions and an understanding of how the helicity content of the nucleon is distributed among its constituents. Eventually these fundamental distributions should be computable using non-perturbative methods such as lattice gauge theory or light-cone hamiltonian diagonalization.

The structure functions discussed in this paper are meant to reflect the intrinsic bound-state structure of the nucleons, and thus they strictly apply only at low resolution scales $Q^2 < Q_0^2$ where QCD evolution can be neglected. They can be used at large Q^2 as the input distributions for perturbative QCD evolution, as in the analyses of Ref. [4]. However, as we discuss below, the actual implementation of the evolution program must take into account the fact that for $x \sim 1$ the bound-state quark which is struck in deep lepton inelastic scattering is far off-shell, thus suppressing its gluon radiation.

The polarized quark and gluon distributions $G_{q/H}(x, \lambda, Q)$ and $G_{g/h}(x, \lambda, Q)$ of a hadron are most simply represented as probability distributions determined by the light-cone wavefunctions $\psi_n(x_i, k_{\perp i}, \lambda_i)$, where $\sum_{i=1}^n x_i = 1$, and $\sum_{i=1}^n k_{\perp i} = 0_\perp$. The square of the invariant mass of an n-particle Fock state configuration in the wavefunction is $\mathcal{M}_n^2 = \sum_{i=1}^n [(k_{\perp i}^2 + m_i^2)/x_i]$. Thus the kinematical regime where one quark has nearly all of the light-cone momentum $x \sim 1$, and the remaining constituents have $x_i \sim 0$, represents a very far off-shell configuration of a bound-state wavefunctions. In the limit $x \to 1$, the Feynman virtuality of the struck parton in a bound state becomes far off-shell and space-like: $k_F^2 - m^2 = x(M_H^2 - \mathcal{M}^2) \to -\mu^2/(1-x)$, where μ is the invariant mass of the system of stopped constituents. If one assumes that the bound-state wavefunction of the hadron is dominated by the lowest invariant mass partonic states, then the constituents can attain far off-shell configurations only by exchanging hard gluons; thus the leading behavior at large virtuality can be computed simply by iterating the gluon exchange interaction kernel [5–7]. This conforms to the usual ansatz of perturbative QCD that hard perturbative contributions dominate amplitudes involving high momentum transfer compared to the contributions arising from non-perturbative sources.

Thus, because of asymptotic freedom, the leading order contributions in $\alpha_s(k_F^2)$ to the quark and gluon distributions at $x \to 1$ can be computed in perturbative QCD from minimally connected tree graphs. For example, in the case of the nucleon structure functions, the dominant amplitude is derived from graphs where the three valence quarks exchange two hard gluons. The tree amplitude is then convoluted with the nucleon distribution amplitude $\phi(x_i, k_F^2)$ which is obtained by integrating the valence three-quark nucleon wavefunction $\psi_3(x_i, k_{\perp i}, \lambda_i)$, over transverse momenta up to the scale k_F^2 [7]. The $dk_\perp d\phi$ azimuthal loop integrations project out only the $L_z = 0$ component of the three-quark nucleon wavefunction. Thus, in amplitudes controlled by the short distance structure of the hadron's valence wavefunction, orbital angular momentum can be ignored, and the valence quark helicities sum to the hadron helicity.

The limiting power-law behavior at $x \to 1$ of the helicity-dependent distributions derived from the minimally connected graphs is

$$G_{q/H} \sim (1-x)^p,$$

where

$$p = 2n - 1 + 2\Delta S_z.$$

Here n is the minimal number of spectator quark lines, and $\Delta S_z = |S_z^q - S_z^H| = 0$ or 1 for parallel or anti-parallel quark and proton helicities, respectively [5]. This counting rule reflects the fact that the valence Fock states with the minimum number of constituents give the leading contribution to structure functions when one quark carries nearly all of the light-cone momentum; just on phase-space grounds alone, Fock states with a higher number of partons must give structure functions which fall off faster at $x \to 1$. The helicity dependence of the counting rule also reflects the helicity retention properties of the gauge couplings: a quark with a large momentum fraction of the hadron also tends to carry its helicity. The anti-parallel helicity quark is suppressed by a relative factor $(1-x)^2$. Similarly, in the case of a splitting function such as $q \to qg$ or $g \to \bar{q}q$, the sign of the helicity of the parent parton is transferred to the constituent with the largest momentum fraction [8]. The counting rule for valence quarks can be combined with the splitting functions to predict the $x \to 1$ behavior of gluon and non-valence quark distributions. In particular, the gluon distribution of non-exotic hadrons must fall by at least one power faster than the respective quark distributions.

The counting rules for the end-point behavior of quark and gluon helicity distributions can also be derived from duality, i.e., continuity between the physics of exclusive and inclusive channels at fixed invariant mass [9]. As shown by Drell and Yan [10], a quark structure function $G_{q/H} \sim (1-x)^{2n-1}$ at $x \to 1$ if the corresponding form factor $F(Q^2) \sim (1/Q^2)^n$ at large Q^2. Recent measurements of elastic electron–proton scattering at SLAC [11] are compatible with the perturbative QCD predictions [12] for both the helicity-conserving $F_1(Q^2)$ and helicity-changing $F_2(Q^2)$ form factors: $Q^4F_1(Q^2)$ and $Q^6F_2(Q^2)$ become approximately constant at large Q^2. The power-law fall-off of the form factors corresponds to the helicity-parallel and helicity-anti-parallel quark distributions behaving at $x \to 1$ as $(1-x)^3$ and $(1-x)^5$, respectively, in agreement with the counting rules. The leading exponent for quark distributions is odd in the case of baryons and even for mesons in agreement with the Gribov–Lipatov crossing rule [13].

The counting rule predictions for the quark and gluon distributions are relevant at low momentum transfer scales $Q_0 \sim \Lambda_{\mathrm{QCD}}$ in which the controlling physics is that of the hadronic bound state rather than the radiative corrections associated with structure function evolution. At the hadronic scale one can normalize the non-singlet quark helicity content of the proton and neutron using the constraint from β decay [14]:

$$\Delta u - \Delta d = \frac{g_A}{g_V} = 1.2573 \pm 0.0028.$$

where $\Delta q_i(x) = q_i^+(x) - q_i^-(x)$, with $i = \mathrm{u, d, s}$, is the difference of the helicity-aligned and helicity-anti-aligned quark distributions in the proton, and $\Delta q_i = \int_0^1 \mathrm{d}x\ \Delta q_i(x)$ is the integrated moment. (In the standard notation, $q^+(x, Q) = G_{\mathrm{q/p}}(x, \lambda_{\mathrm{q}} = \lambda_{\mathrm{p}}, Q) + G_{\bar{\mathrm{q}}/\mathrm{p}}(x, \lambda_{\mathrm{q}} = \lambda_{\mathrm{p}}, Q)$ so that both quark and anti-quark contributions are included.) In addition, if one assumes SU(3) flavor symmetry, hyperon decay also implies a polarized strange quark component in the proton wavefunction [15,16],

$$\frac{\Delta u + \Delta d - 2\Delta s}{\sqrt{3}} = 0.39.$$

Thus only one normalization is left undetermined.

The presence of polarized gluons in the nucleon wavefunction implies that polarized strange quarks contribute to the nucleon helicity-dependent structure functions at some level. There is also evidence from neutrino–proton elastic scattering that the proton has a significant polarized strange quark content [15]. Our parameterization of the polarized strange quark distributions with a significant helicity fraction $\Delta s \simeq 0.10$ and a small momentum fraction $\langle x_s \rangle \simeq 0.035$. The shape represents the sum of contributions from radiatively generated $\mathrm{s} + \bar{\mathrm{s}}$ quarks as well as intrinsic strange quarks, intrinsic to the nucleon bound states.

The helicity-dependent structure function $g_1(x, Q^2)$ measured in deep inelastic polarized-lepton–polarized-proton scattering can be identified in the Bjorken scaling region with the quark helicity asymmetry:

$$g_1(x, Q^2) = \tfrac{1}{2} \sum_q e_{\mathrm{q}}^2 \Delta q(x, Q^2).$$

The first moment of the proton–neutron difference has zero anomalous dimension and satisfies the Bjorken sum rule [8]:

$$\int_0^1 \mathrm{d}x [g_1^{\mathrm{p}}(x, Q^2) - g_1^{\mathrm{n}}(x, Q^2)] = \frac{1}{6} \frac{g_A}{g_V} \left(1 - \frac{\alpha_s(Q^2)}{\pi} + \ldots \right),$$

where the last factor represents the radiative corrections from hard gluon interactions in the electron–quark scattering process. [2] Thus the QCD radiative corrections [3] to the

[2] For recent analyses of the radiative corrections to the Bjorken sum rule see Ref. [17].

[3] An analysis of the evolution of the helicity-dependent quark and gluon structure functions is given in Ref. [18].

helicity-dependent structure functions can modify the shape of the distributions, within the global constraint of the Bjorken sum rule.

At high Q^2, the radiation from the struck quark line increases the effective power-law fall-off $(1-x)^p$ of structure functions relative to the underlying quark distributions: $\Delta p = (4C_F/\beta_1)\log[\log(Q^2/\Lambda^2)/\log(Q_0^2/\Lambda^2)]$, where $C_F = \frac{4}{3}$ and $\beta_1 = 11 - \frac{2}{3}n_f$. The counting rule predictions for the power p thus provide a *lower bound* for the effective exponent of quark structure functions at high $Q^2 > Q_0^2$. However, in the end-point region $x \sim 1$, the struck quark is far off-shell and the radiation is quenched since one cannot evolve Q^2 below $Q_0^2 \simeq k_f^2 = -[\mu^2/(1-x)]$, the Feynman virtuality of the struck parton [19]. Furthermore, the integral of the g_1 structure function is only affected by QCD radiative corrections of order $\alpha_s(Q^2)/\pi$.

Thus PQCD can give useful predictions for the power-law fall-off of helicity-aligned and anti-aligned structure functions at $x \sim 1$. Higher order contributions involving additional hard gluon exchange are suppressed by powers of $\alpha_s(k_F^2)$. Further iterations of the interaction kernel will give factors of fractional powers of $\log(1-x)$ analogous to the anomalous dimensions $\log^{\gamma_n} Q^2$ which appear in the PQCD treatment of form factors at large momentum transfer [12]. This is in contrast to super-renormalizable theories such as QCD(1 + 1) where the power-law behavior in the end-point region is modified by all-order contributions [20].

The fact that one has a definite prediction for the $x \sim 1$ behavior of leading twist structure functions is a powerful tool in QCD phenomenology, since any contribution that does not decrease sufficiently fast at large x is most likely due to coherent multi-quark correlations. As discussed in Ref. [21], such contributions are higher twist, but they arise naturally in QCD and are significant at fixed $(1-x)Q^2$. Such coherent contributions are in fact needed in order to explain the anomalous change in polarization seen in pion-induced continuum lepton-pair and hadronic J/ψ production experiments at high x_F [22].

At large x the perturbative QCD analysis predicts "helicity retention" – i.e., the helicity of a valence quark with $x \sim 1$ will match that of the parent nucleon. This result is in agreement with the original prediction of Farrar and Jackson [6] that the helicity asymmetry $\Delta q(x)$ approaches 1 at $x \to 1$. We also predict, in agreement with Ref. [6], that the ratio of unpolarized neutron to proton structure functions approaches the value $\frac{3}{7}$ for $x \to 1$.

In the following sections we will analyze the shape of the polarized gluon and quark distributions in the proton. First we will study the behavior of the gluon asymmetry $\Delta G(x)/G(x)$ (polarized over unpolarized distributions) at small values of x, where it turns out to be proportional to x with a coefficient approximately independent of the details of the bound-state wavefunction. We then write down a simple model for the gluon distributions which incorporates the counting rule constraints at $x \to 1$. The same is done for the up, down and strange quark distributions. The extrinsic and intrinsic strange quark distributions are also discussed, paying special attention to the inclusive–exclusive connection with the strange quark contribution to the proton form factors.

2. Helicity-dependent gluon distributions

The angular momentum of a fast-moving proton has three sources: the angular momentum carried by the quarks, the angular momentum carried by the gluons, and the orbital angular momentum carried by any of the constituents. Angular momentum conservation for J_z at a fixed light-cone time implies the sum rule [23]

$$\tfrac{1}{2}(\Delta u + \Delta d + \Delta s) + \Delta G + \langle L_z \rangle = \tfrac{1}{2}. \tag{2.1}$$

Here $\Delta G \equiv \int_0^1 \mathrm{d}x\, \Delta G(x)$ is the helicity carried by the gluons, where $\Delta G(x)$ is the difference between the helicity-aligned and anti-aligned gluon distributions $G^+(x)$ and $G^-(x)$; the unpolarized gluon distribution $G(x)$ is the sum of these two functions, $G(x) \equiv G^+(x) + G^-(x)$. The corresponding definitions for the quark distributions are $\Delta q(x) = q^+(x) - q^-(x)$ and $q(x) = q^+(x) + q^-(x)$ with $q =$ u, d, s. By definition, the anti-quark contributions are included in $\Delta q(x)$ and $q(x)$. As emphasized by Ma [24], the helicity distributions measured on the light-cone are related by a Wigner rotation (Melosh transformation) to the ordinary spins S_i^z of the quarks in an equal-time rest-frame wavefunction description. Thus, due to the non-collinearity of the quarks, one cannot expect that the quark helicities will sum simply to the proton spin.

In this paper we shall present model forms for the gluon distribution functions $\Delta G(x)$ and $G(x)$ for nucleons which incorporate the known large-x counting rule constraints:

$$G^+(x) \to C(1-x)^4 \quad (x \to 1), \tag{2.2}$$

$$G^-(x) \to C(1-x)^6 \quad (x \to 1). \tag{2.3}$$

We will also incorporate a constraint on the behavior of the gluon asymmetry ratio $\Delta G(x)/G(x)$ for small x:

$$\left(\frac{\Delta G(x)}{G(x)}\right)_{\text{proton}} \to \frac{x}{3}\left\langle \frac{1}{y} \right\rangle \quad (x \to 0). \tag{2.4}$$

This last theoretical constraint will be demonstrated below. Here $\langle 1/y \rangle$ stands for the first inverse moment of the quark light-cone momentum fraction distribution in the proton lowest Fock state. For this state we expect $\langle 1/y \rangle \simeq 3$.

A simple form for baryon gluon distributions, which incorporates the limiting behaviors presented above, is

$$\begin{aligned} \Delta G(x) &= \frac{N}{x}\left[1-(1-x)^2\right](1-x)^4, \\ G(x) &= \frac{N}{x}\left[1+(1-x)^2\right](1-x)^4. \end{aligned} \tag{2.5}$$

In this model the momentum fraction carried by the gluons in the proton is $\langle x_g \rangle \equiv \int_0^1 \mathrm{d}x\, xG(x) = \frac{12}{35}N$, and the helicity carried by the gluons is $\Delta G \equiv \int_0^1 \mathrm{d}x\, \Delta G(x) = \frac{11}{30}N$. Taking the momentum fraction $\langle x_g \rangle$ to be $\frac{1}{2}$, we predict $\Delta G = 0.54$.

Such large values for the gluon momentum fraction are inconsistent with the assumption that the proton has a dominant three-quark Fock state probability; a self-consistent approach thus requires taking into account gluon radiation from the full quark and gluon light-cone Fock basis of the nucleon. Our main emphasis here is to predict the characteristic shapes of the polarized quark and gluon distributions. The large-x regime is clearly dominated by the lowest particle-number Fock states. We thus expect the qualitative features of the model to survive in a more rigorous approach; in particular, it is apparent from the structure of the model that the gluon helicity fraction will be of the same order of magnitude as the gluon momentum fraction.

The prediction that $\Delta G \simeq 0.5$ is phenomenologically interesting. If one also accepts the experimental suggestion from EMC that the quark helicity sum $\Delta u + \Delta d$ is small, then this implies that gluons could carry a significant fraction of the proton helicity $J_z = \frac{1}{2}$ of the same size as the momentum fraction carried by the gluons. However, one also expects significant orbital angular momentum L_z which arises, for example, from the finite transverse momentum associated with the $q \to qg$ gluon emission matrix element.

We now proceed to prove Eq. (2.4) for the low-x behavior of the asymmetry $\Delta G(x)/G(x)$. In this region the quarks in the hadron radiate coherently, and we must consider interference between amplitudes in which gluons are emitted from different quark lines. An analysis of this type was first presented in Ref. [25], and in this note we extend and correct some of the results of that paper.

As an example, we first analyze the helicity content of positronium, where we can ignore internal transverse momenta and non-collinearity. Consider the ortho-positronium two-fermion $J_z = 1$ Fock state in which the particles have helicities $++$. Following the calculation of Ref. [25], we obtain

$$\left(\frac{\Delta G(x)}{G(x)}\right)_{\mathrm{ortho}(J_z=+1)} \simeq x\left\langle\frac{1}{y}\right\rangle \simeq 2x \quad (x \to 0). \tag{2.6}$$

In the case of para-positronium (and also for $J_z = 0$ ortho-positronium), in which we start with a Fock state with helicities $+-$, the result is $\Delta G(x) = 0$. This is because for every diagram in $G^+(x)$ there is a corresponding diagram in $G^-(x)$, but with the helicities of all the particles reversed.

We now apply a similar analysis to the gluon distribution in the nucleon. We start with a three-quark Fock state in which the quarks have helicities $+++$ as would be appropriate for the helicity content of an isobar state Δ with $J_z = \frac{3}{2}$. Then the result found in Ref. [25], i.e.

$$\left(\frac{\Delta G(x)}{G(x)}\right)_{\Delta(J_z=3/2)} \simeq x\left\langle\frac{1}{y}\right\rangle, \tag{2.7}$$

follows.

In the nucleon case, however, we start with a three-quark Fock state with helicities $++-$. Thus clearly there is a cancellation between the squared terms in which the gluon is emitted from one of the positive helicity quarks versus the contributions in which the gluon is emitted by a negative helicity quark. The interference terms work similarly, ensuring a finite result for both $G(x)$ and $\Delta G(x)$ at zero $k_\perp$, just as in the case of photon distributions in positronium. Then the positive helicity quarks have a dominant $G^+(x)$ and contribute positively to $\Delta G(x)$; similarly, the negative helicity quarks contribute negatively to $\Delta G(x)$. To see this more clearly, consider the photon emitted by a single electron with $J_z = +\frac{1}{2}$. Then $G^+_{\gamma/e}(x) = 1/x$ and $G^-_{\gamma/e}(x) = (1-x)^2/x$. Thus $\Delta G(x)/G(x) = x$ at $x \to 0$ with unit coefficient in this case. The sign reverses for an electron with $J_z = -\frac{1}{2}$.

The generated gluon asymmetry distribution in the nucleon at low x is then given by Eq. (2.4). The extra factor of $\frac{1}{3}$ is due to the fact that all the quarks contribute positively to $G(x)$, but they give contributions proportional to the sign of their helicity in $\Delta G(x)$. The main assumption setting the value of the gluon asymmetry at $x \to 0$ is the estimated value of the inverse moment $\langle 1/y \rangle$. For realistic wavefunctions this expectation value may receive very large (possibly divergent) contributions from near $y = 0$. However, one must be careful at this point because in deriving Eq. (2.4) we assumed that $x \ll y$. In order to be consistent with this assumption we will perturb around a constituent quark wavefunction which is strongly peaked around $y = \langle y \rangle = \frac{1}{3}$. We have furthermore assumed for simplicity that $\langle y \rangle$ is the same for all valence quarks, although this is inconsistent with results from QCD sum rules [26]. (One could improve the estimate for $\langle 1/y \rangle$ by allowing for different momentum fractions for the helicity-up and helicity-down quarks. This would evidently reduce ΔG, since it is known that $\langle y \rangle$ is larger for helicity-parallel quarks. Furthermore, in QCD we expect that higher Fock states will contribute to reduce the value of $\langle y \rangle$ away from $\frac{1}{3}$, which would be the expected value if only the three-quark valence Fock state was present.)

3. Helicity-dependent quark distributions

In this section we shall construct a simple polynomial model for the helicity-dependent quark distributions in the proton and neutron.

As we have discussed in the previous sections, at $x \sim 1$ PQCD predicts that the helicity-parallel quark distribution $q^+(x)$ is enhanced relative to the helicity-anti-parallel quark distribution $q^-(x)$ by two powers of $1-x$. The property of helicity retention at large x is a direct consequence of the gauge theory couplings between quarks and gluons. For the valence quarks in a nucleon the counting rules predict

$$q^+(x) \sim (1-x)^3 \quad (x \to 1) \tag{3.1}$$

and

$$q^-(x) \sim (1-x)^5 \quad (x \to 1). \tag{3.2}$$

The case of the non-valence strange quarks is somewhat more complex and will be discussed in detail in the next section. The result is

$$s^+(x) \sim (1-x)^5 \quad (x \to 1), \tag{3.3}$$

$$s^-(x) \sim (1-x)^7 \quad (x \to 1). \tag{3.4}$$

For $x \sim 0$ the helicity correlation disappears since the constituent has infinite rapidity $\Delta y = \log x$ relative to the nucleon's rapidity.

The strange quark distribution in a nucleon can arise from both intrinsic and extrinsic contributions. The intrinsic contribution is associated with the multiparticle Fock state decomposition of the hadronic wavefunction, and it is essentially of non-perturbative origin. This is in contrast to the extrinsic component, which arises from $s\bar{s}$ pair production from a gluon emitted by a valence quark and is associated with the self-field of a single quark in the proton. From evolution and gluon splitting, the extrinsic strange contributions are known to behave as

$$s_e^+(x) \sim (1-x)^5 \quad (x \to 1), \tag{3.5}$$

$$s_e^-(x) \sim (1-x)^7 \quad (x \to 1). \tag{3.6}$$

The Drell–Yan inclusive–exclusive connection relates the high-Q^2 behavior of the hadronic form factors to the large-x limit of the quark distribution functions; i.e.

$$F(Q^2) \xrightarrow{Q^2 \to \infty} \frac{1}{(Q^2)^n} \Leftrightarrow G_{q/p} \xrightarrow{x \to 1} (1-x)^{2n-1+2\Delta S_z}, \tag{3.7}$$

where $\Delta S_z = 0$ or 1 for parallel or anti-parallel quark and proton helicities, respectively. If we naively apply this prescription to the extrinsic strange quark component, we would predict that the strange quark contribution to the electromagnetic proton form factor should fall as $1/Q^6$, since in this case $n = 3$. But a direct calculation of the strange quark contribution to either the axial or vector form factor of the nucleon gives only a nominal $1/Q^4$ behavior, which is the same power-law fall-off as the valence quark contribution. In the leading order calculations the loop integrals connecting a hard $s\bar{s}$ loop to a valence quark all have momenta $l = O(Q)$, thus producing radiative corrections of order $\alpha_s^N(Q)$, to the exclusive amplitude with $N = 2$ (axial) or $N = 3$ (vector), rather than extra powers of $1/Q^2$ [7]. The solution to this apparent contradiction is that we should apply the inclusive–exclusive connection for the strange quark contributions to a transition form factor connecting an initial state with three quarks (uud) to a final state in which a strange pair has been created (uuds$\bar{s}$), as in the transition form factor $p \to \Lambda K$, at *fixed* final-state mass. Since the internal hard-scattering matrix element T_H for (uud) + $\gamma^* \to$ sudu$\bar{s}$ has three off-shell fermion legs, this transition form factor falls off as $(1/Q^2)^3$, and it correctly satisfies the inclusive–exclusive connection ($n = 3$).

One can also consider the case where Q^2 and the final-state mass are both large, but there is a K and Λ in the final state. This again corresponds to a $\sim (1-x)^5$ structure function. In the case of the transition $p \to p\phi$, there is a color mismatch in T_H at lowest order. Thus this amplitude should be suppressed (Zweig rule) by an extra power of $\alpha_s(Q^2)$. Of course all of this holds for the analogous charm systems as well.

The intrinsic strange components are associated with Fock states having at least five particles; the distributions thus have the behavior

$$s_i^+(x) \sim (1-x)^7 \quad (x \to 1), \tag{3.8}$$

$$s_i^-(x) \sim (1-x)^9 \quad (x \to 1), \tag{3.9}$$

which corresponds to $n=4$ in the spectator quark counting rules. It also satisfies the inclusive–exclusive connection, since the intrinsic contribution to the form factor falls as $(1/Q^2)^4$.

For the complete parameterization we shall adopt the canonical forms:

$$u^+(x) = \frac{1}{x^\alpha}\left[A_u(1-x)^3 + B_u(1-x)^4\right],$$

$$d^+(x) = \frac{1}{x^\alpha}\left[A_d(1-x)^3 + B_d(1-x)^4\right],$$

$$u^-(x) = \frac{1}{x^\alpha}\left[C_u(1-x)^5 + D_u(1-x)^6\right],$$

$$d^-(x) = \frac{1}{x^\alpha}\left[C_d(1-x)^5 + D_d(1-x)^6\right],$$

$$s^+(x) = \frac{1}{x^\alpha}\left[A_s(1-x)^5 + B_s(1-x)^6\right],$$

$$s^-(x) = \frac{1}{x^\alpha}\left[C_s(1-x)^7 + D_s(1-x)^8\right],$$

where we require

$$A_q + B_q = C_q + D_q$$

to ensure the convergence of the helicity-dependent sum rules. Thus in our model, the Regge behavior of the asymmetry $\Delta q(x) \sim x^{-\alpha_R}$ is automatically one unit less than the unpolarized intercept: $\alpha_R = \alpha - 1$. Isospin symmetry at low x (Pomeron dominance) also requires

$$A_u + B_u + C_u + D_u = A_d + B_d + C_d + D_d.$$

We emphasize that these distributions include both the quark and anti-quark contributions.

Our parameterization of the helicity-dependent quark distributions is close in spirit to the parameterization D_0' for the unpolarized quark and gluon distributions given by Martin, Roberts, and Stirling [27]. The MRS parameterizations are a good match to our unpolarized forms $q(x) = q^+(x) + q^-(x)$ since the MRS forms combine counting rule constraints with a good fit to a wide range of perturbative QCD phenomenology. We find that choosing the effective QCD Pomeron intercept $\alpha = 1.12$ allows a good match to the unpolarized quark distributions given by the MRS parameterization D_0' at $Q^2 = 4$ GeV^2 over the range $0.001 < x < 1$. It also predicts an increasing structure function $F_2(x, Q^2)$ for $x < 10^{-3}$, as suggested in the recent data from HERA [28]. Thus we predict $\alpha_R = 0.12$ for the helicity-changing reggeon intercept. The momentum fraction carried by the quarks (and anti-quarks), $\langle x_q \rangle = \int_0^1 dx\, xq(x)$, where $q(x) \equiv q^+(x) + q^-(x)$, is assumed to be ~ 0.5.

A combined analysis [15] of the SLAC and EMC [29,30] polarized electron–proton data provides the constraint

$$\int dx\, g_1^p(x) = 0.112 \pm 0.009 \pm 0.019.$$

If one uses the central value together with the constraints from nucleon and hyperon decay and includes radiative corrections of $O((\alpha_s/\pi)^3)$ then one obtains the following values for the proton helicity carried by the different quarks [15].

$$\Delta u = 0.83 \pm 0.03, \quad \Delta d = -0.43 \pm 0.03, \quad \Delta s = -0.10 \pm 0.03, \tag{3.10}$$

at the renormalization scale $Q^2 = 10\ \mathrm{GeV}^2$. Since these values for the Δq are obtained after removing the deep inelastic radiative corrections, we can use them as the initial phenomenological inputs for the proton; the neutron distributions then follow from isospin symmetry. The small value for the total quark helicity $\Delta\Sigma = \Delta u + \Delta d + \Delta s = 0.31 \pm 0.07$ is consistent with large N_C predictions in QCD [31], and it is about half of the value $\Delta\Sigma \simeq 0.75$ expected in the relativistic three-quark constituent model for the nucleon without dynamical gluons [32]. As we shall show below, the gluon helicity fraction Δg scales closely with the gluon momentum fraction $\langle x_g \rangle$.

The $u(x)$ and $d(x)$ parameterizations have eight parameters which we will fix using the following eight conditions: three conditions arise from the requirement that the sum rules converge at $x \to 0$; two conditions come from the values of Δu and Δd; one condition follows by imposing the SU(6) large-x relation $A_u = 5A_d$; one condition is obtained from the empirical value of the Gottfried sum $S_G \equiv \int dx \frac{1}{3}[u(x) - d(x)] = 0.235$ [33]; the final condition is obtained from the sum of momentum fractions carried by the quark and anti-quark, $x_u + x_d = 0.521$ [27]. It is straightforward to find parameters for the polynomial forms which are consistent with the above inputs:

$$A_u = 3.784, \quad A_d = 0.757, \tag{3.11}$$

$$B_u = -3.672, \quad B_d = -0.645, \tag{3.12}$$

$$C_u = 2.004, \quad C_d = 3.230, \tag{3.13}$$

$$D_u = -1.892, \quad D_d = -3.118. \tag{3.14}$$

With this set of parameters, the respective quark momentum fractions are

$$\langle x_u \rangle = 0.331, \quad \langle x_d \rangle = 0.190. \tag{3.15}$$

The predicted distributions $xu(x)$, $xd(x)$, $\Delta d(x)$, and $\Delta u(x)$ are shown in Figs. 1a and 1b. In each case both the quark and anti-quark contributions are included. The simple polynomial forms represent a simple parameterization of the non-perturbative polarized and unpolarized quark distributions which satisfy the known theoretical constraints at large and small x and the empirical sum rules. We also show a comparison of the unpolarized distributions with the MRS D_0' phenomenological parameterizations. The agreement is quite reasonable. The differences in the shapes of the distributions can be attributed to the effects of perturbative QCD evolution.

Notice that $\Delta d(x) \equiv d^+(x) - d^-(x)$ is positive at large x (which follows from $A_u = 5A_d$), and negative at small to moderate values of x. We thus predict that $\Delta d(x)$ will change sign and go through zero at some physical value for x. With the above parameterization the zero of $\Delta d(x)$ occurs at $x = 0.489$.

In the case of the strange quark plus strange anti-quark distributions, we have four parameters and three conditions: one from the convergence of sum rules; one from the

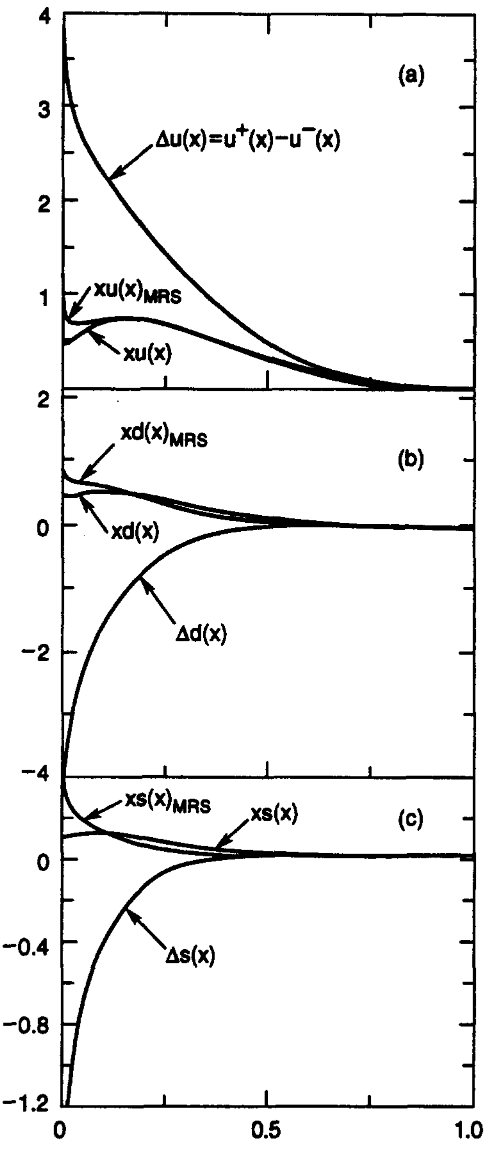

Fig. 1. Model predictions for the non-perturbative polarized $\Delta q(x) = q^+(x) - q^-(x)$ and unpolarized quark $xq(x) = x[q^+(x) + q^-(x)]$ distributions inthe proton. The polynomial forms satisfy sum rule and dynamical constraints. The leading Regge behavior at $x \to 0$ has the intercept $\alpha = 1.12$. By definition both quark and antiquark contributions are included. Comparison with the MRS D_0' parameterization for the unpolarized quark distributions [27] are also shown. (a) $u(x)$ distributions, (b) $d(x)$ distributions, (c) $s(x)$ distributions.

value of Δs; and one from the momentum fraction carried by strange plus anti-strange quarks $x_s = 0.035$ [34]. This leaves us with one unknown, which we choose to C_s. The three constraints give the solution set

$$A_s = -0.6980 + 0.9877 C_s, \quad B_s = 0.8534 - 1.1171 C_s,$$
$$D_s = 0.1551 - 1.1294 C_s. \tag{3.16}$$

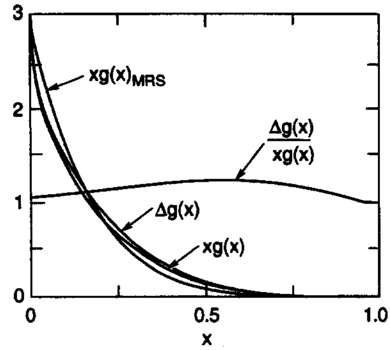

Fig. 2. Predictions for the non-perturbative polarized $\Delta G(x) = G^+(x) - G^-(x)$ and unpolarized gluon $xG(x) = x[G^+(x) + G^-(x)]$ distributions in the proton. The polynomial forms satisfy sum rule and dynamical constraints. The leading Regge behavior at $x \to 0$ has the intercept $\alpha_g = 1.12$. Comparison with the MRS D_0' parameterization for the unpolarized gluon distributions [27] is also shown.

Because of the probabilistic interpretation of parton distribution functions, $s^+(x)$ and $s^-(x)$ must both be non-negative functions of x, which implies the rather stringent bounds

$$0.7067 < C_s < 1.2013.$$

Within these bounds, $g_1(x)$ is practically independent of C_s; to be definite, we chose $C_s = 1$. (We could have taken any other value consistent with the inequalities [4].) We compare our simple parameterization to the MRS D_0' parameterization in Fig. 1c. The MRS distribution which gives an approximate realization of the data rises faster at low x than our model. This could be attributed to the need to impose a higher Pomeron intercept, or the the effects of QCD evolution.

We can also find parameterizations for the polarized gluon distributions which are consistent with the $x \to 0$ and $x \to 1$ helicity constraints, as well as the MRS unpolarized gluon distribution:

$$G^+(x) = \frac{1}{x^{\alpha_g}}\left[A_g(1-x)^4 + B_g(1-x)^5\right], \tag{3.17}$$

$$G^-(x) = \frac{1}{x^{\alpha_g}}\left[A_g(1-x)^6 + B_g(1-x)^7\right]. \tag{3.18}$$

This form automatically incorporates the coherence constraint, Eq. (2.4). We shall assume that $\alpha_g = \alpha = 1.12$ so that the pomeron intercept is identical for quark and gluon distributions. The parameters set $A_g = 2$ and $B_g = -1.25$ gives an unpolarized gluon distribution $G(x) = G^+(x) + G^-(x)$ similar to the phenomenological D_0' gluon distribution given by MRS (see Fig. 2). The momentum carried by the gluons in the nucleon using the above simple form is $\langle x_g \rangle = 0.42$. (The gluon and light quark and anti-quark distributions then almost saturate the momentum sum rule.) The gluon helicity content

[4] For an alternative parameterization of the strange quark distributions, see Ref. [35].

for the above parameterization is $\Delta G = 0.45$. As shown in the figure, the shape of the polarized distribution $\Delta G(x)$ given by the above parameterization is almost identical to $xG(x)$.

Alternatively, if we take $\alpha_g = 1$, then the parameter set $A_g = 0.2381$ and $B_g = 1.1739$ again gives the same values $\langle x_g \rangle = 0.42$ and $\Delta G = 0.45$ as above. In this case, the resulting shape unpolarized distribution $G(x) = G^+(x) + G^-(x)$ is indistinguishable from the phenomenological D'_0 gluon distribution given by MRS.

Although there is some experimental information about the unpolarized gluon distribution, this is not the case for the polarized gluon distribution. It is important to test these distributions directly, for example in processes such as J/ψ production in polarized ep and pp collisions [35].

4. Polarized structure functions

In this section we will use the polynomial model forms for $\Delta q(x)$ and $q(x)$ to compute the polarized helicity structure functions of nucleons:

$$g_1^{\mathrm{ep}}(x) = \tfrac{1}{2}\left[\tfrac{4}{9}\Delta u(x) + \tfrac{1}{9}\Delta d(x) + \tfrac{1}{9}\Delta s(x)\right] \tag{4.1}$$

and

$$g_1^{\mathrm{en}}(x) = \tfrac{1}{2}\left[\tfrac{4}{9}\Delta d(x) + \tfrac{1}{9}\Delta u(x) + \tfrac{1}{9}\Delta s(x)\right], \tag{4.2}$$

and compare the results to the recent experiments. (Note that $\Delta q(x)$ refers to the combined asymmetries from both quarks and anti-quarks in the proton.) A precise prediction requires the inclusion of QCD evolution. Here we will, as in Ref. [15], simply include the normalization factor $N_{\mathrm{QCD}} = 1 - \alpha_s/\pi \approx 0.92$ arising from QCD radiative corrections. The Bjorken sum rule for the difference of proton and neutron quark helicities is automatically satisfied. The Ellis–Jaffe sum rule for the nucleon quark helicity is violated by the model due to the presence of the strange quark contributions Δs.

We have emphasized that the dynamics of QCD implies helicity retention: the quark with x close to 1 has the same helicity as the proton. Thus all of the structure function asymmetries become maximal at $x \to 1$, and the ratio of unpolarized proton and neutron structure functions can be predicted.

According to the standard SU(6) flavor and helicity symmetry, the probabilities to find u and d quarks of different helicities in the proton's three-quark wavefunction are: $P(\mathrm{u}^+) = \frac{5}{9}$, $P(\mathrm{d}^+) = \frac{1}{9}$, $P(\mathrm{u}^-) = \frac{1}{9}$, $P(\mathrm{d}^-) = \frac{2}{9}$ [37]. Thus the usual expectation from SU(6) symmetry is $F_2(\mathrm{n})/F_2(\mathrm{p}) = \frac{2}{3}$ for all x. As Farrar and Jackson pointed out [6], this naive SU(6) result cannot apply to the local helicity distributions since the helicity-aligned and helicity-anti-aligned distributions have different momentum distributions. At large x, u^- and d^- can be neglected relative to u^+ and d^+, and thus SU(6) is broken to $\mathrm{SU(3)}^+ \times \mathrm{SU(3)}^-$. Our model retains the SU(6) ratio $P(\mathrm{u}^+) : P(\mathrm{d}^+) = A_{\mathrm{u}} : A_{\mathrm{d}} = 5:1$ at large x, so that we predict $F_2(\mathrm{n})/F_2(\mathrm{p}) \to \frac{3}{7}$ as $x \to 1$. The physical

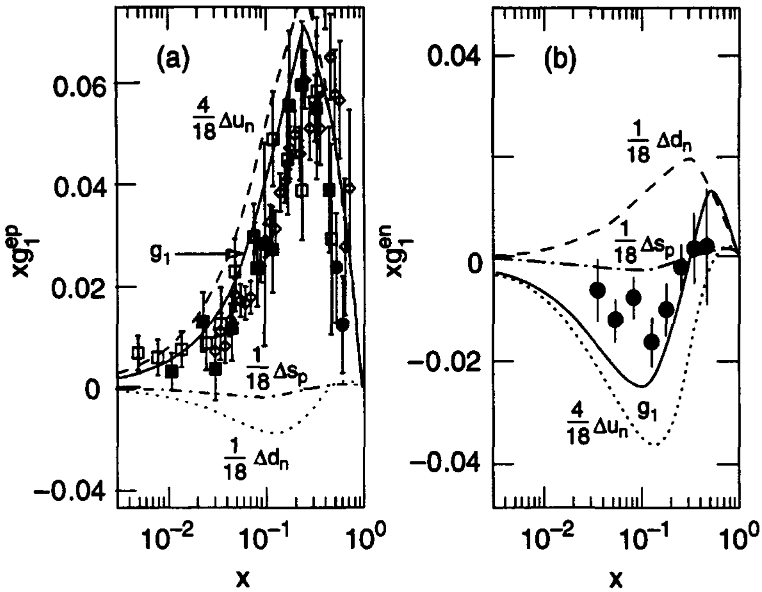

Fig. 3. (a) Model prediction for the polarized helicity structure function of the proton compared with experiment. Full line: sum of all flavors; dashed: only up quarks; dotted: only down quarks; dash-dotted: only strange quarks. We have multiplied our prediction with a PQCD correction factor $1-\alpha_s/\pi = 0.92$. The data are from SLAC EMC (closed circles), EMC (closed squares), SMC (open squares) and SLAC E143 (diamonds). (b) Same as (a) but for the neutron. The data are from the SLAC E142 experiment [2].

picture that emerges is that the struck quark carries all the helicity of the nucleon, and the spectators have $S_z = 0$, although their total helicity is a combination of 0 and 1. This wavefunction is just a piece of the full SU(6) wavefunction, but since it is the piece that contains the u^+ and d^+, and since this part remains unchanged, the ratio $P(u^+):P(d^+)$ is still $5:1$.

Notice that the only empirical input into our model are the integrated values of the various flavors obtained from the proton data. The shape of the polarized distributions is essentially determined by the perturbative QCD constraints. The agreement with the shape of the SLAC and EMC experimental data for the proton is rather good (see Fig. 3a) and could be further improved by taking into account PQCD evolution.

We can also compare our model with the polarized neutron structure function extracted by the E142 from its polarized-electron–polarized-He^3 measurements (see Fig. 3b). For the neutron we predict two new effects which are not present in the proton. First g_1^{en} tends to fall faster than g_1^{ep} for large x. This is because as in the Carlitz–Kaur [38] and Farrar–Jackson [6] models, the helicity-aligned up quark dominates the proton distribution and the helicity down quark dominates the neutron structure function at large x. A related effect is that $g_1^{en}(x)$ changes sign as a function of x. This is due to the fact that except for large x (where the helicity-aligned down quark dominates) g_1^{en} is dominated by the anti-aligned up quark distribution. Since $\int_0^1 dx\, \Delta u_n(x) = \int_0^1 dx\, \Delta d_p(x) < 0$ [15], it is clear that $g_1^{en}(x)$ must be negative at small x.

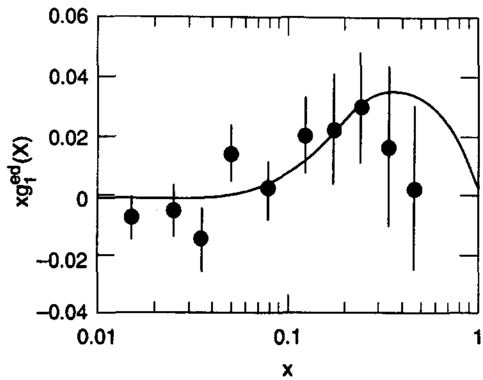

Fig. 4. Polarized helicity structure function of the deuteron. The data are from Ref. [1]. We have multiplied our prediction from the sum of proton and neutron contributions by a D-state depolarization factor $1-\frac{3}{2}\omega_D$, with $\omega_D = 0.058$, and the PQCD correction factor $1-\alpha_s/\pi = 0.92$.

A comparison of our model with the recent SMC data for the polarized deuteron structure function $g_1^d(x)$ is shown in Fig. 4. The shape of the data appears to be consistent with our predictions, except possibly at the largest-x point where the model predicts too little asymmetry. To make this prediction, we have, as in Ref. [1], assumed that the deuteron structure function is half of the sum of the neutron and proton structure functions and included the D-state depolarization factor with D-state probability 0.058. The model then predicts the normalization

$$\int \mathrm{d}x\, g_1^d(x) = \tfrac{1}{2}\int \mathrm{d}x[g_1^p(x) + g_1^n(x)]$$
$$= \left(\tfrac{5}{36}(\Delta u + \Delta d) + \tfrac{1}{18}\Delta s\right)\left(1 - \frac{\alpha_s}{\pi}\right)\left(1 - \tfrac{3}{2}\omega_D\right) = 0.038,$$

compared to the SMC result

$$\int \mathrm{d}x\, g_1^d(x) = 0.023 \pm 0.020(\text{stat.}) \pm 0.015(\text{syst.}).$$

The distributions presented in this paper have applicability to any PQCD leading twist process which requires polarized quark and gluon distributions as input. Our input parameters have been adjusted to be compatible with global parameters of available current experiments. The values can be refined as further and more precise polarization experiments become available. A more precise parameterization should also take into account corrections from QCD evolution, although this effect is relatively unimportant for helicity-dependent distributions. Our central observation is that the shape of the distributions is almost completely predicted when one employs the constraints obtained from general QCD arguments at large x and small x.

A remarkable prediction of our formalism are the very strong correlations between the parent hadron helicity and each of its valence quark, sea quark, and gluon constituents at large light-cone momentum fraction x. Although the total quark helicity content of the proton is small, we predict a strong positive correlation of the proton's helicity with the helicity of its u quarks and gluon constituents. The model is also consistent with the assumption that the strange plus anti-strange quarks carry 3.5% of the proton's momentum and -10% of its helicity. We also note that completely independent predictions based on QCD sum rules also imply that the three-valence-quark light-cone distribution amplitude has a very strong positive correlation at large x when the u quark and proton helicities are parallel [26].

Acknowledgements

We would like to thank Peter Lepage, Al Mueller, Michael Peskin, Felix Schlumpf, and Wai-Keung Tang for helpful conversations.

References

[1] B. Adeva et al., Phys. Lett. B 302 (1993) 533.
[2] P.L. Anthony et al., Phys. Rev. Lett. 71 (1993) 959;
K. Abe et al., preprint SLAC-PUB-6508 (1994);
For recent reviews see R.G. Milner, Plenary talk at SPIN-94, MIT preprint LNS 94-88 (1994);
S. Forte, preprint CERN-TH.7453/94, *in* Proc. Tennessee Int. Symp. on Radiative corrections, Gatlinburg (1994), to be published.
[3] D. Adams et al., Phys. Lett. B 329 (1994) 399.
[4] M. Gluck, E. Reya and A. Vogt, Phys. Lett. B 306 (1993) 391, and references therein.
[5] R. Blankenbecler and S.J. Brodsky, Phys. Rev D 10 (1974) 2973;
J.F. Gunion, Phys. Rev. D 10 (1974) 242;
S.J. Brodsky and G.P. Lepage, *in* Proc. 1979 Summer Inst. on Particle physics, SLAC (1979).
[6] G.R. Farrar and D.R. Jackson, Phys. Rev. Lett. 35 (1975) 1416.
[7] G.P. Lepage and S.J. Brodsky, Phys. Rev. D 22 (1980) 2157.
[8] J.D. Bjorken, Phys. Rev. D 1 (1970) 1376.
[9] E. Bloom and F. Gilman, Phys. Rev. Lett. 25 (1970) 1140.
[10] S.D. Drell and T.-M. Yan, Phys. Rev. Lett. 24 (1970) 181.
[11] A. Lung et al., Phys. Rev. Lett. 70 (1993) 718.
[12] S.J. Brodsky and G.P. Lepage, *in* Perturbative quantum chromodynamics, ed. A.H. Mueller (World Scientific, Singapore, 1989).
[13] V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 438, 675.
[14] Particle Data Group, Review of particle properties, Phys. Rev. D 45 (1992) Part II.
[15] J. Ellis and M. Karliner, preprint CERN-TH-7324/94; Phys. Lett. B 313 (1993) 131;
See also R.L. Jaffe and A. Manohar, Nucl. Phys. B 321 (1989) 343;
F.E. Close, R.G. Roberts, Phys. Lett. B 316 (1993) 165;
S.D. Bass, A.W. Thomas, Cavendish preprint HEP-93-4 (1993).
[16] M. Bourquin et al., Z. Phys. C 21 (1983) 27.
[17] J. Ellis and M. Karliner, preprint CERN-TH-7324/94; Phys. Lett. B 313 (1993) 131;
S.J. Brodsky and H.J. Lu, preprint SLAC-PUB-6481 (1994), Phys. Rev. D., to be published.
[18] E.L. Berger and J. Qiu, Phys. Rev. D 40 (1989) 3128.
[19] For further discussion, see S.J. Brodsky and G.P. Lepage, *in* Proc. 1979 Summer Inst. on Particle physics, SLAC (1979).
[20] M.B. Einhorn, Phys. Rev. D 14 (1976) 3451.
[21] S.J. Brodsky, P. Hoyer, A.H. Mueller and W. Tang, Nucl. Phys. B 369 (1992) 519.
[22] S.J. Brodsky, *in* Proc. 10th Int. Symp. on High-energy spin physics, Nagoya, Japan (1992).
[23] See, for example, R.L. Jaffe and A. Manohar, Nucl. Phys. B 337 (1990) 509.

[24] B.Q. Ma, Z. Phys. C 58 (1993) 479.
[25] S.J. Brodsky and I.A. Schmidt, Phys. Lett. B 234 (1990) 144;
Coherence effects are also discussed in S.J. Brodsky and J.F. Gunion, Phys. Rev. D 19 (1979) 1005;
Applications to atomic and molecular systems are discussed in M. Burkardt, Nucl. Phys. B 373 (1992) 371;
see also M. Kaluža, A.G. Schneider-Neureither and H. Pirner, Universität Heidelberg preprint (1993).
[26] V.L. Chernyak and A.R. Zhitnitsky, Phys. Rep. 112 (1984) 173.
[27] A.D. Martin, W.J. Stirling and R.G. Roberts, Phys. Lett. B 306 (1993) 145; preprints RAL-94-055, HEP-PH-9406315.
[28] ZEUS Collaboration, Phys. Lett. B 315 (1993) 481.
[29] M.J. Alguard et al., Phys. Rev. Lett. 37 (1976) 1261; 41 (1978) 70;
G. Baum et al., Phys. Rev. Lett. 51 (1983) 1135.
[30] J. Ashman et al., Phys. Lett. B 206 (1988) 364; Nucl. Phys. B 328 (1989) 1.
[31] See, for example, S.J. Brodsky, J. Ellis and M. Karliner, Phys. Lett. B 206 (1988) 309.
[32] S.J. Brodsky and F. Schlumpf, Phys. Lett. B 329 (1994) 111.
[33] NMC Collaboration, Phys. Rev. D 50 (1994) 1;
P. Amaudruz et al., Phys. Rev. Lett. 66 (1991) 2712.
[34] CCFR, Phys. Rev. Lett. 70 (1993) 134.
[35] G. Preparata, P.G. Ratcliffe and J. Soffer, Phys. Lett. B 273 (1991) 306.
[36] T. Morii, S. Tanaka and T. Yamanishi, Phys. Lett. B 322 (1994) 253;
see also A.V. Efremov, L. Mankiewicz and N.A. Tornqvist, Phys. Lett. B 284 (1992) 394.
[37] See, for example, W.M. Gibson and B.R. Pollard, *in* Symmetry principles in elementary particle physics (Cambridge U.P., Cambridge, 1976), p. 330.
[38] R.D. Carlitz and J. Kaur, Phys. Rev. Lett. 38 (1977) 673;
J. Kaur, Nucl. Phys. B 128 (1977) 219;
For recent empirical models, see E.L. Berger and J. Qiu, Phys. Rev. D 40 (1989) 3128;
K. Kobayakawa et al., Phys. Rev. D 46 (1992) 2854.